PION 2013 - Exercises

1 A diabolo as a yo-yo (12 points)

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A diabolo consists of two cones, connected tip-to-tip by a cilinder-shaped connector piece (see figure 1.1). The connector has a outer diameter D and its mass can be neglected. Both cones have a base with radius R and height H. The mass density $\rho(x)$ of both cones linearly increases to the top: $\rho(x) = \left(\frac{|x|+H}{H}\right)\rho_0$.

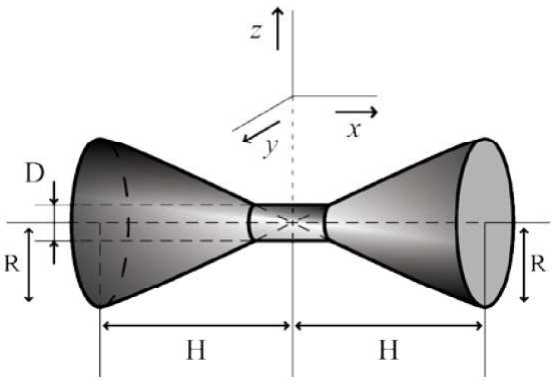

Figure 1.1: A diabolo, consisting of two cones, connected by a cilinder-shaped connector.

A rope of neglectable mass and diameter with length L is winded around the connector. One side of the rope is held fixed at height L above the ground. The diabolo is dropped and rolls towards the ground along the rope. At $t = 0$ the diabolo hits the ground. At that moment the diabolo is spinning, but has no horizontal speed, so it is slipping completely. Because of friction with the ground, the diabolo will start to translate as well as rotate. From $t = T_1$ onwards, the diabolo is rolling without slipping. The gravitational acceleration is g and the coefficient of kinetic friction is μ_k.

Question 1: Calculate the moment of inertia I of the diabolo around the x-axis. Express the anwser in terms of the mass of the diabolo and the radius R.

Question 2: Calculate the angular velocity of the rotation of the diabolo at $t = 0$. Express the anwser in terms of the given quantities.

Now, assume that the diabolo is spinning with angular velocity ω_0 at moment $t = 0$.

Question 3: Calculate the distance the diabolo covers before it stops slipping at $t = T_1$. Express the anwser in terms of the given quantities.

Question 4: Calculate the work done by the force of friction over the distance covered between $t = 0$ and $t = T_1$. Express the anwser in terms of the given quantities.