5. Flow in a pipe

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8 points

Consider a long, round pipe with water flowing through. The inner radius of the pipe is $R = 0.025m$. The length of the pipe is $L = 1km$. We presume that the flow through the pipe is stationary, i.e. that the velocity profile of the water in the pipe isn't changing as a function of time nor as a function of the coordinate in the flow direction. The velocity profile $u(r)$ is purely a function of the radius r, where the radius is measured from the axis of the pipe. The variable u depicts the velocity of water. The pressure drop over the pipe length L is represented by variable Δp.

- Beneath a critical value the flow will be laminar. Solving the theoretical equations of flow (Navier-Stokes, 1822) will give a parabole as velocity profile. The speed of water will be zero near the side ($r = R$), and maximal at the axis ($r = 0$).
- Above a critical value the flow will be turbulent. The turbulent velocity profile is often described with the so called 'power-law', where the velocity profile is proportional with $(Y/R)^c$. Here y is the distance to the side and c is a constant (say $c = \frac{1}{7}$.
- The critical value is given by the so called Reynolds number $Re = \frac{\rho\ u_{ave} D}{\mu}$, named after the famous British scientist Osborne Reynolds (1842-1912). Here ρ is the density of water, D is the inner diameter of the pipe and μ is the dynamic viscosity of water.

The average speed of water by the tube cross section is given by u_{ave}. The transition from a laminar flow to a turbulent flow usually takes place at a critical Reynolds number of approximately $Re \approx 2300$.

Water often behaves like a Newtonian fluid. When that's the case, the viscose shear stress τ, which is exercised by the water on the side of the pipe, is given by the dynamic viscosity of water multiplied by the velocity gradient of the water at the side of the pipe, $\tau = \mu\ \frac{du}{dr}|_{r=R}$.

Questions

1. Derive an expression for the loss of pressure Δp along the pipe as a function of the shear stress τ at the side of the wall.

2-a. Construct a graph of this pressure drop along the pipe as a function of the average velocity of water, u_{ave}, with the reach of velocity $u_{ave} \in [0, 0.1] ms^{-1}$. Use a BINAS (a Dutch book with important properties of elements and basic compostions) to find the correct properties of water.

(Hint: In order to determine the shear stress, one has to determine the shape of the velocity profile as a function of the radius r).

2-b. Why is it impossible to construct the entire graph?

3-a. An emperical formula of turbulent pressure drop along the pipe Δp is given by:

$$\Delta p = \frac{0.3164}{(Re)^{0.25}} \frac{L}{2R} \frac{1}{2} \rho u_{ave}^2 \tag{5.1}$$

What's the graph now?

3-b. What's striking about this graph?