1. Maxwell's insight

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8 points

Maxwell's laws are as follows:

- electrostatica:
  - $\oint_{kring} \vec{E} \cdot d\vec{l} = 0$
  - $\Phi_E \equiv \oint_{oppervlak} \vec{E} \cdot d\vec{\sigma} = \frac{Q^{omsloten}}{\epsilon_0}$
- magnetostatica:
  - $\oint_{kring} \vec{B} \cdot d\vec{l} = \mu_0 I^{omsloten}$
  - $\Phi_B \equiv \oint_{oppervlak} \vec{B} \cdot d\vec{\sigma} = 0$

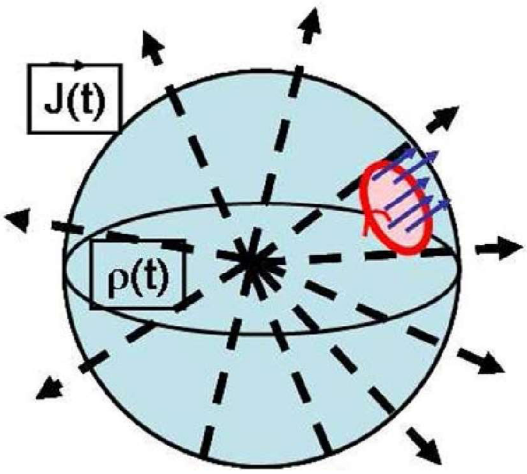

Figure 1.1: The amount of charge in the origin

In the origin of our coordinate system, we find an amount of electrical charge: $Q(t) = Q_0 e^{\frac{-t}{\tau}}$ where t is the time, τ is a constant and Q_0 is the charge at time $t = 0$.

1. Calculate, using Gauss' law, the electrical field that is induced by the charge in the origin. Indicate with a drawing which 'Gaussian surface' you've used.

The electrical charge in the origin can't just disappear. We presume that the charge moves away (flows) in a radial direction.

2. Give an expression for the electric current $I(t)$ and the density current $\vec{J}$. (Be aware that the density current is a vector and therefore, it has a direction.)

3. Argument, with use of a symmetry argument, there can't be a magnetic field parallel to the surface of the imaginary ball in figure 1.1.

4. Argument, if necessary with the Ampère's circuital law, that there's a magnetic field parallel to the surface of the ball. You could be inspired by the Ampère loop which is included in figure 1.1.

The answers given in **3.** and **4.** are, as you might have noticed, contradictionary. Luckily, we all know that Maxwell solved this contradiction in a brilliant fashion, by adding a term to Ampère's circuital law.

5. Give Ampère's circuital law as an integral, with Maxwell's addition. Show that, with this extra term, the magneticfield at **4.** will disappear as well. (Hint: use answers found at **1.** and **2.**)