4 Ultrashort electron pulses (13 points)

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Electron microscopy and electron diffraction are two techniques to investigate the composition and structure of materials. The last few years, researchers are trying to achieve ultrafast time resolution as well as atom-scale resolution. To this end, short, intense electron pulses of high quality ('brightness') are used. However, due to electronic repulsion it is hard to pack a large number of electrons in a small space. In this exercise, we investigate the properties of uniform, ellipsoidal packages of electrons, i.e. electron clouds with constant charge density inside a ellipsoid.

Consider an ellipsoidal charge cloud descriped in Cartisian coordinates by

$$\frac{x^2+y^2}{A^2}+\frac{z^2}{C^2}\leq 1, \tag{4.1}$$

where A and C are constant lenghts. Within the cloud, the charge density ρ is constant, given by ρ_0, and outside the cloud the charge density is zero. According to Maxwells laws, the electrostatic potential V satisfies

$$\nabla^2 V = -\rho/\epsilon_0,$$

and from V we can calculate the electric field $\vec{E}$.

Question 1: Show that inside the cloud the electric field depends linearily on one coordinate. Sketch the direction and size of the electric field on the boundary of the cloud in the xz-plane in the case $C < A$.

Question 2: Sketch the electric field for points on the x-axis, both inside and outside the cloud. Sketch in the same figure an intuitive approximation of the electric field for the same points in the case that the charge distribution is not uniform but given by a Gaussian:

$$\rho(x,y,z)=\rho_0\exp\left[-\frac{1}{2}\left(\frac{x^2+y^2}{A^2}+\frac{z^2}{C^2}\right)\right].$$

Question 3: The cloud of electrons will quickly disintegrate under influence of the internal electric field. Show that the uniform ellipsoidal electron packages nonetheless always stay uniform and ellipsoidal (the *aspect ratio* C/A can change). To simplify things, assume that at time $t = 0$ the electron cloud is of the form of equation 4.1, and that all electron are motionless. *Hint:* Show that an electron at starting position (x_0, y_0, z_0) will satisfy $x(t) = x_0 X(t)$ (with $X(t)$ a function only depending on the time t) and something similar for $y(t)$ and $z(t)$. Explain why this implies that an ellipsoidal cloud stays ellipsoidal.

Creating uniform ellipsoidal electron clouds in free space in hard. However, it is possible using lasers to free a surface charge density $\sigma(x, y)$ on the surface of a flat photocathode (for example a flat sheet of copper) and to then draw the charge from the plate using an acceleration field. If $\sigma(x, y)$ is chosen right, this charge will evolve into a uniform ellipsoidal electron cloud. The right choice of the charge distribution comes down to creating an extremely flat uniform ellipsoidal pulse, i.e., an ellipsoidal cloud according to 4.1 with lengths C' and A' with $0 \approx C' \ll A'$. We assume the surface charge density on the cathode is zero outside of $x^2 + y^2 = R^2$ (so $A' = R$) and that it contains the same total charge as the cloud in equation 4.1.

Question 4: Give an expression of $\sigma(x, y)$ in terms of x, y, R, A, C and ρ_0.

The quality of the electron pulse is characterised by its so-called *emittance* ϵ: the smaller the emittance, the better the pulse can be focused on a sample, for example using electrostatic lenses. For the x-emittance the following expression holds:

$$\epsilon^2 = \langle x^2 \rangle \langle v_x^2 \rangle - \langle x v_x \rangle^2,$$

where v_x denotes the speed of an electron in the x-direction, and $\langle \cdots \rangle$ denotes an average over the cloud density. To be clear: $\langle x^2 \rangle$ is the variance of the position of an electron. On time $t = 0$, the emittance of the ideal uniform ellipsoidal electron cloud in question 3 is equal to zero because all charge is stationary.

Question 5: Show that for this ideal cloud the emittance is zero for each time t. (This implies that uniform ellipsoidal electron clouds have almost ideal properties for electron-optical systems.)

5 Pour some sugar on me (6 points)

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Someone pours from some height, with constant flow, sugar on a scale. At a given moment, the scale indicates exactly 1 kg, and at that moment, he stops pouring.

Question: When all sugar that was still on its way down has come to rest on the scale, what does the scale indicate? You can neglect friction and assume the suger is uniformely accelerated, without collisions.